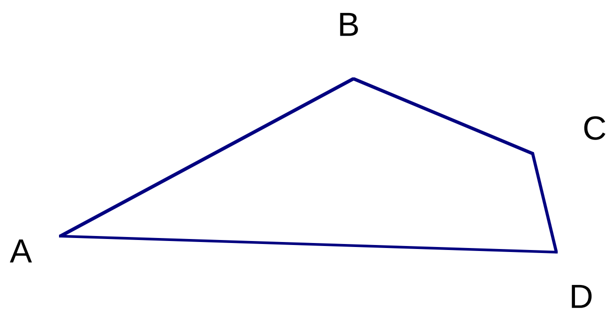
Middle School Proof Assessment

- administered to approximately 450 6th-8th grade students
- each student responded to a subset of the 15 items during a 45 minute class period

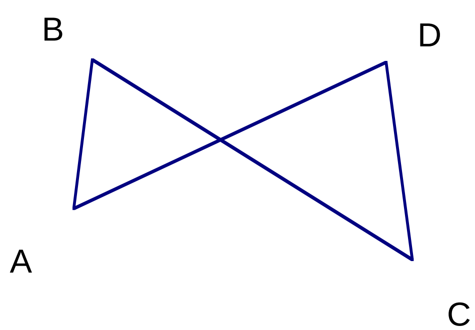
Enzo, a student from Banneker Middle School, comes up with the following definition for a quadrilateral:

A quadrilateral is what you get if you take four points *A, B, C,* and *D* and join them with four straight lines.

Enzo says the following figure is a quadrilateral:



Carla, a classmate of Enzo, says the following figure is also a quadrilateral.



According to Enzo's definition, has Carla drawn a quadrilateral? Why or why not?

Mark each of the following statements true (T) or false (F). Explain each answer.

_____ 1. A quadrilateral that has four equal sides must be a square.

Explain your answer.

_____ 2. A quadrilateral that has four right angles must be a square.

Explain your answer.

_____ 3. A quadrilateral with three equal sides and four right angles must be a square.

Explain your answer.

The following two statements are mathematical facts:

Diagonals in rectangles always have the same length.
Some rhombuses are rectangles.

Based on the statements above, check (√) the <u>one</u> statement that you think is true:

Diagonals in rhombuses never have the same length. ______

Diagonals in rhombuses sometimes have the same length. ______

Diagonals in rhombuses always have the same length. ______

Explain your choice.

Mei discovers a number trick. She takes a number and multiplies it by 5 and then adds 12. She then subtracts the starting number and divides the result by 4. She notices the answer she gets is 3 more than the number she started with.

For example, suppose Mei starts with 7.	**7**
She would multiply by 5 to get 35	**7 x 5 = 35**
Then she adds 12 to get 47.	**35 + 12 = 47**
She subtracts the original number, 7, to get 40	**47 – 7 = 40**
Then she divides 40 by 4 to get 10.	**40 ÷ 4 = 10**

Ten is three more than **7**, *her starting number.*

Malaika doesn't think this will happen again. So she tries the trick with another number.

For example, suppose Malaika starts with 10.	**10**
She would multiply by 5 to get 50.	**10 x 5 = 50**
She then adds 12 to get 62.	**50 + 12 = 62**
She subtracts the original number, 10, to get 52.	**62 – 10** = **52**
She then divides 40 by 4 to get 13.	**52 ÷ 4 = 13**

Thirteen is three more than **10**, *her starting number.*

Mei and Malaika decide that they will always get a result that is three more than the starting number.

Do you think they are right?

How would you convince a classmate that you would always get a result that is three more than the starting number?

Three students are discussing whether the following statement is <u>always</u> true.

The sum of any three consecutive whole numbers is equal to three times the middle number. For example, 4, 5 and 6 are consecutive numbers and 4 + 5 + 6 equals 15, which equals three time the middle number, 5.

Their explanations are shown below.

Marian says:

Suppose that n-1, n, and n+1 are 3 consecutive numbers.
(n-1) + n + (n+1) = 3n

So the sum of the three numbers is three times the middle number, n.

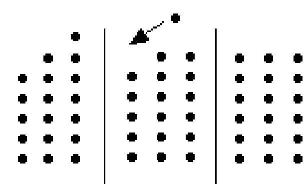
So Marian says it's true.

Jamie says:

I found a way using marbles. I make three columns containing five, six, and seven marbles. I then move the top marble from the highest column and move it to the lowest column, making the number of marbles in each column equal. Look at my drawing at the right to see what I mean.

So the total number of marbles is three times the number in the middle column.

So Jamie says it's true.



Jamie's drawing

Jeff says:

5 + 6 + 7 = 3 x 6
7 + 8 + 9 = 3 x 8
569 + 570 + 571 = 3 x 570

Thus, the sum of three consecutive numbers is always three times the middle number.

So Jeff says it's true.

Refer to the problems on the previous page to answer the following questions.

Whose answer do you like the best? Explain why.

Whose answer do you understand the best? Explain why.

Whose answer do you think would be most convincing to classmates? Explain why.

Whose answer do you think would get the highest grade from the teacher? Explain why.

For each student's response, circle the answer you think is best.

Marian's answer:	agree	don't know	disagree
Shows that the statement is <u>always</u> true	1	2	3
Only shows that the statement is <u>sometimes</u> true	1	2	3
Shows you <u>why</u> the statement is true	1	2	3
Is an <u>easy way to explain</u> to a classmate who is unsure	1	2	3

Jamie's answer:	agree	don't know	disagree
Shows that the statement is <u>always</u> true	1	2	3
Only shows that the statement is <u>sometimes</u> true	1	2	3
Shows you <u>why</u> the statement is true	1	2	3
Is an <u>easy way to explain</u> to a classmate who is unsure	1	2	3

Jeff's answer:	agree	don't know	disagree
Shows that the statement is <u>always</u> true	1	2	3
Only shows that the statement is <u>sometimes</u> true	1	2	3
Shows you <u>why</u> the statement is true	1	2	3
Is an <u>easy way to explain</u> to a classmate who is unsure	1	2	3

The sum of two consecutive numbers is always an odd number. For example,* 5 + 6 = 11 *and* 8 + 9 = 17. *Show that the sum of any two consecutive numbers is always an odd number.

How do you know when something is right in mathematics?

What does it mean to prove something in mathematics?

The following statement is true in mathematics:

If two even numbers are multiplied, then their product is even.

Based on the statement above, do you think the following statements are true or false.

______ 1. If 32 and 186 are multiplied, then the product will be even.

Explain your answer.

______ 2. 286 is even, so it is the product of two even numbers.

Explain your answer.

______ 3. If I multiply 68 by some even number, it’s possible to get a product that is odd.

Explain your answer.

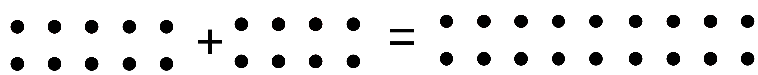
Four students came up with different ways to show that the following statement is always true.

When you add any two even numbers, your answer is always even.

Their explanations are shown below.

Yvonne's answer

Yvonne says, "I can draw any even number using pairs of dots."



(10) (8) (18)

Yvonne says that this diagram would work for any two even numbers because you would only be adding pairs of dots.

So Yvonne says it's true.

Bonnie's answer

2 + 2 = 4	4 + 2 = 6
2 + 4 = 6	4 + 4 = 8
2 + 6 = 8	4 + 6 = 10

So Bonnie says it's true.

Duncan's answer.

Even numbers end in 0, 2, 4, 6, or 8.

When you add any two of these the answer will still end in 0, 2, 4, 6, or 8.

So Duncan says it's true.

Eric's answer

Let x = any whole number, y = any whole number

$x + y = z$

$z - x = y$

$z - y = x$

$z + z - (x + y) = x + y + 2z$

So Eric says it's true.

Refer to the problems on the previous page to answer the following questions.

Whose answer do you like the best? Explain why.

Whose answer do you understand the best? Explain why.

Whose answer do you think would be most convincing to classmates? Explain why.

Whose answer do you think would get the highest grade from the teacher? Explain why.

For each student's response, circle the answer you think is best.

Bonnie's answer:	agree	don't know	disagree
Shows that the statement is always true	1	2	3
Only shows that the statement is sometimes true	1	2	3
Shows you why the statement is true	1	2	3
Is an easy way to explain to a classmate who is unsure	1	2	3

Eric's answer:	agree	don't know	disagree
Shows that the statement is always true	1	2	3
Only shows that the statement is sometimes true	1	2	3
Shows you why the statement is true	1	2	3
Is an easy way to explain to a classmate who is unsure	1	2	3

Duncan's answer:	agree	don't know	disagree
Shows that the statement is always true	1	2	3
Only shows that the statement is sometimes true	1	2	3
Shows you why the statement is true	1	2	3
Is an easy way to explain to a classmate who is unsure	1	2	3

Yvonne's answer:	agree	don't know	disagree
Shows that the statement is always true	1	2	3
Only shows that the statement is sometimes true	1	2	3
Shows you why the statement is true	1	2	3
Is an easy way to explain to a classmate who is unsure	1	2	3

The following two statements are mathematical facts:

Diagonals in kites are always perpendicular.
A rhombus is a kite.

Based on the statements above, check (√) the <u>one</u> statement that you think is true:

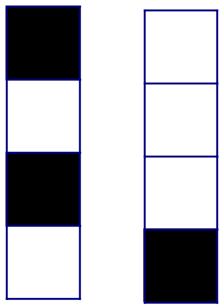
Diagonals in rhombuses are never perpendicular. _____

Diagonals in rhombuses are sometimes perpendicular. _____

Diagonals in rhombuses are always perpendicular. _____

Explain your choice.

Imagine that you have a set of black and white blocks. Your teacher asked you to build towers that are four blocks high. Two examples of such a tower are given below.



1. **How many different towers that are four blocks high can you make, using any combination of black and white blocks?** (Use the space below to show your work and CIRCLE your answer.)

2. **How will you know when you have them all?**

Amy and Stephan are trying out a number trick.

Amy picks a number between 1 and 10.	**Amy's number:**	**6**
She adds it to 10 and writes down the answer.	**Step # 1:**	**10 + 6 = 16**
She takes the starting number away from 10 and writes down the answer.	**Step # 2:**	**10 – 6 = 4**
Then she adds the two answers from the two steps.	**Final Answer:**	**16 + 4 = 20**

Stephan picks a number between 1 and 10.	**Stephan's number:**	**3**
He adds it to 10 and writes down the answer.	**Step # 1:**	**10 + 3 = 13**
He takes the starting number away from 10 and writes down the answer.	**Step # 2:**	**10 – 3 = 7**
Then he adds the two answers from the two steps.	**Final Answer:**	**13 + 7 = 20**

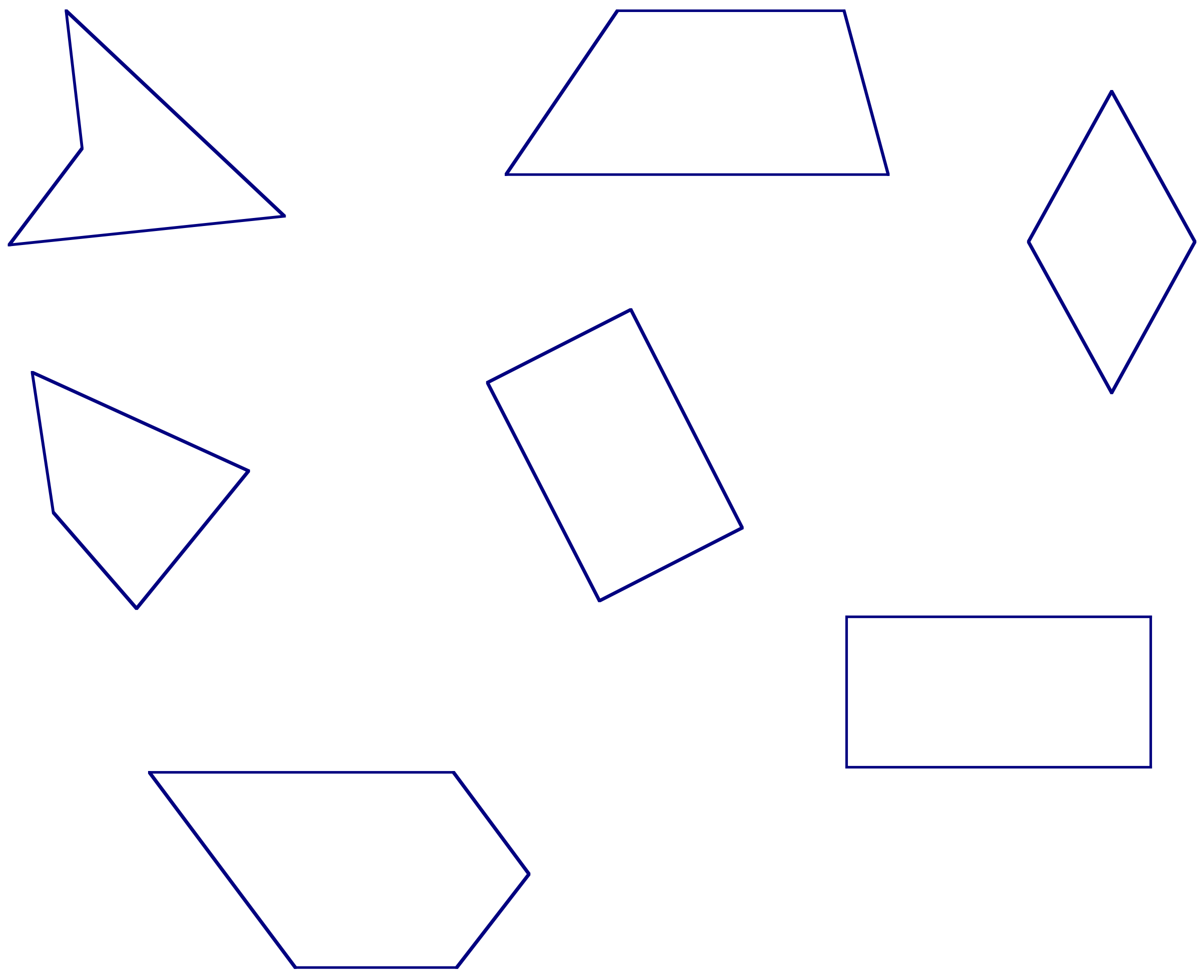
What do you notice about the two final answers?

Will you always get the same final answer no matter what your starting number is?

How would you convince a classmate that you would always get the same answer?

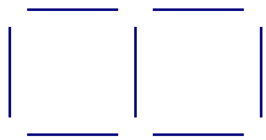
Definition: A parallelogram is a quadrilateral whose opposite sides are parallel. For example, a square is a parallelogram because both pairs of its opposite sides are parallel.

Circle all the figures below that are parallelograms.

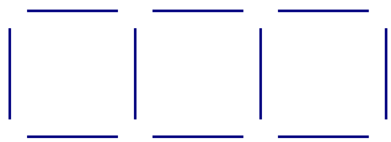


Show that when you add any two even numbers, your answer is always even. Provide an explanation that would convince a classmate that the answer is always even.

If a side of each square in the shape below is 1 toothpick, then it takes 7 toothpicks to make 2 squares in a row.

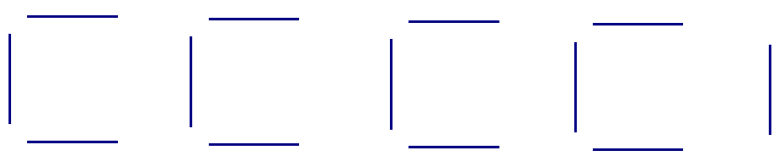


How many toothpicks are required to make 3 squares in a row?



Did you count 10?

A student in another class says that she can figure out the number of toothpicks needed for any number of squares in a row. For example, suppose we have 4 squares in a row and want to figure out how many toothpicks we need. She says that she thinks of each square starting with 3 toothpicks shaped like a "C" and then 1 toothpick is used at the very end.



To figure out the number of toothpicks needed to build 4 squares in a row, we take 3 times the number of squares and then add 1 for the toothpick at the very end. **She writes out her rule this way: 3 • 4 + 1 = 13.** So it takes 13 toothpicks.

Please answer the questions on the next page.

Use her rule to figure out how many toothpicks it would take to build 6 squares in a row.

Do you think her rule will always work?

How do you know?

References

Item A1: Galbraith, P. (1995). Mathematics as reasoning. *Mathematics Teacher*, *88*(5), 412-417.

Item B9: Healy, L. & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for Research in Mathematics Education*, *31*(4), 396-428.

Item B11: Maher, C. & Martino, A. (1996). The development of the idea of mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education, 27*(2), 194-214.

Item B12: Bell, A. (1976). A study of pupils' proof-explanations in mathematical situations. *Educational Studies in Mathematics, 7,* 23-40.