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# A PULSATION PHASE–DEPENDENT DUST SHELL MODEL OF OH 26.5+0.6

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## ABSTRACT

We have modeled the spectral energy distribution of the radio-luminous OH/IR star 26.5+0.6 as a function of pulsation phase using standard radiative transfer techniques but with more self-consistent input parameters based partly on pulsation model calculations. The changes in spectral shape and overall intensity are easily explained in terms of the radial movement of the dust condensation radius with the changing luminosity of the star. The derived amplitude of the bolometric light curve is only 1.2 mag, considerably less than expected.

*Subject headings:* infrared: sources — stars: circumstellar shells — stars: individual (OH 26.5+0.6) — stars: long-period variables — stars: pulsation

## I. INTRODUCTION

Many of the radio-luminous OH/IR (RLOHIR) stars are losing mass at rates of up to $10^{-4}$ $M_{\odot}$ $yr^{-1}$ (e.g., Jones 1987; Herman and Habing 1985). The mechanism driving the mass loss appears to be a combination of pulsation-driven shocks and radiation pressure on dust grains. This view is based on the study of classical Mira variables as well as the heavily obscured RLOHIR stars. The shocks elevate gas far beyond the photosphere where it becomes cool enough for dust grains to form. Once dust forms, radiation pressure on the grains accelerates the material to terminal velocity (Jones, Ney, and Stein, 1981; Bowen 1988*a*, *b*).

Mira variables typically vary by more than 5 mag at visual wavelengths with periods of a few hundred days. Their bolometric luminosity, most of it in the near-infrared, varies much less. The RLOHIR stars typically vary with larger bolometric amplitudes and longer periods than the optical Mira variables (Engels *et al.* 1983; Jones 1987). The RLOHIR stars, then, provide the theorist with more extreme pulsation characteristics than the classical Mira variables. The optically thick circumstellar shells of these stars make observations of the photosphere nearly impossible except at wavelengths near 2 $\mu$m (Jones *et al.* 1987). As a consequence, the emergent energy distribution of the circumstellar shell must be the focus of theoretical modelling.

To date, most modeling of the cirumstellar shells around the RLOHIR stars concentrate on radiative transfer dust shell models. These models employ a combination of ad hoc and empirical grain characteristics and density distributions along with a healthy dose of guided intuition (e.g. Rowan-Robinson and Harris 1983; Volk and Kwok 1988). In most cases, the models are computed for static conditions. Rowan-Robinson and Harris (1983) modeled a few variable stars at different phases, but the stars they chose did not show extreme variability, and each phase was modeled independently. Self-consistent dust shell models taking into account the variability of the central star have yet to be constructed.

Usually the density distribution of the shell is considered to be a free parameter in the modeling, with some restrictions, of course. Recently, theoretical models of the pulsation phenomena in the circumstellar environment of Mira variables have begun to provide predictions for the density and velocity distribution of gas and dust above the photosphere of the star. These theoretical density distributions provide a guide to the density distribution to be used in dust shell models.

In this paper, we explore the application of standard dust shell model techniques in combination with pulsation model results to the extreme variable RLOHIR star OH 26.5+0.6. This star varies by more than a factor of 10 at 3.5 $\mu$m and has a very long but very regular pulsation period of 1570 days (Jones 1987; Jones *et al.* 1989). Forrest *et al.* (1978) observed 26.5+0.6 in the 10 $\mu$m band at two different pulsation phases and found that the depth of the 10 $\mu$m silicate absorption feature changed with phase. The combination of 10 $\mu$m spectra, a physically realistic density and velocity distribution, an excellent light curve, and the extreme nature of its pulsation characteristics make 26.5 + 0.6 an ideal choice to test our understanding of the dust shells around evolved stars.

Our goal is to model the emergent spectral energy distrubiton of OH 26.5+0.6 as a function of phase in as self-consistent a manner as possible. In § II we define the optical properties of the dust we will use in the model. In § III we discuss the theoretical pulsation models developed by Bowen. In § IV we use the information from §§ II and III to model 26.5 + 0.6 at several phases in its pulsation cycle, discuss the significance of the model, and explore future directions for research.

## II. DUST PARAMETERS

A fundamental knowledge of the optical properties of circumstellar dust is lacking. It has been known for some time that pure terrestrial silicates, so-called clean silicates, are incapable of reproducing the strong 1–5 $\mu$m continuum observed in dusty oxygen-rich giants. Various attempts to add more strongly absorbing materials such as iron and carbon have been tried with some success (e.g., Mitchell and Robinson 1978). We chose to use the approach taken by Jones and Merrill (1976) and determine the absorptive component of the index of refraction based partly on fitting the observed flux distribution from 2 to 5 m and partly on the optical properties of terrestrial silicates, particularly at 10 and 20 $\mu$m. Our adopted dust parameters are very nearly the same as the so-called dirty silicates of Jones and Merrill at most wavelengths, with some modifications around 20 $\mu$m.

Day (1976*a*, *b*) finds that the extinction efficiency of silicate material is dependent on temperature. When cooled to temperatures below about 200 K, the absorption efficiency at 20 $\mu$m increases while the absorption at 10 $\mu$m remains essentially unchanged. For M giants with relatively thin dust shells and the silicate feature in emission, most of the dust will be at a high enough temperature that adjustments to the 20 $\mu$m opacity will be unnecessary. For the heavily enshrouded OH/IR stars, however, considerable dust will be at temperatures below 200 K. Ideally we would let the 20 $\mu$m dust opacity be a function of temperature in the circumstellar shell, but this is beyond the scope of this work. Instead, we slightly increased the 20 $\mu$m opacity everywhere in the shell to $Q_{ext}(20\ \mu m)/Q_{ext}(10\ \mu m) = 0.7$ compared to the value of 0.5 used by Jones and Merrill (1976) and Volk and Kwok (1988).

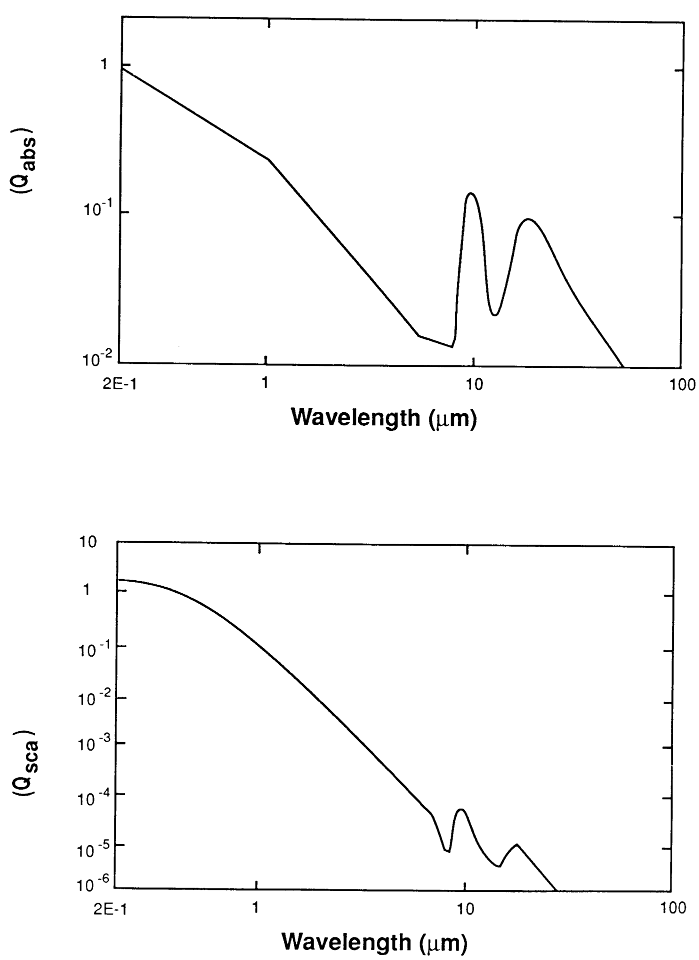


FIG. 1.—Absorption and scattering efficiencies for the grains used in the dust shell model. The grain size distribution was modeled by a Gaussian centered at 0.1 $\mu$m with a width of 0.05 $\mu$m.

TABLE 1

ADOPTED PROPERTIES OF OH 26.5+0.6 AT MINIMUM PHASE

| Property | Value |
|---|---|
| Luminosity | $1 \times 10^4\ L_\odot$ |
| Temperature | 2000 K |
| Radius | 850 $R_\odot$ |
| Mass | 1.4 $M_\odot$ |
| Period | 1570 days |

The wavelength dependence of the absorption and scattering efficiencies for the dust grains used in our models is shown in Figure 1. We use a Gaussian distribution of grain sizes centered at 0.1 $\mu$m with a dispersion of 0.05 $\mu$m and a minimum grain size of 0.01 $\mu$m. A Gaussian distribution is chosen because the vaporization of grains near the star when it is rising in luminosity will tend to destroy the smaller grains, tending to peak the grain size distribution. As it turns out, acceptable model results can be obtained with a more traditional power-law distribution of grain sizes, indicating the dust shell model is relatively insensitive to the exact form of the grains size distribution.

Studies of dust condensation around classical novae (Gehrz 1988) suggest that grains condense on time scales of several weeks, much shorter than the pulsation periods for RLOHIR stars. Although a complete theory for grain condensation is lacking, the work of Yamamoto and Hasegawa (1977) provides a useful theoretical means of assessing the time scales involved in grain formation in the expanding circumstellar shells. We obtain the scale parameter $X = \tau_{sat}/\tau_{coll}$ from Kozasa, Hasegawa, and Seki (1984) for a stellar luminosity of $1 \times 10^{-5}\ L_\odot$ and a mass loss rate of $5 \times 10^{-5}\ M_\odot\ yr^{-1}$ (see Table 1). The saturation time is computed to be about 1 day based on the work of Deguchi (1980) for a condensation radius of 7000 $R_\odot$, corresponding to a dust condensation temperature of 1000 K. Figure 2 shows the results of the calculations for nuclei formation rate and growth of grains using these input parameters appropriate to the environment of RLOHIR stars (Suh 1988). The final grain radius (with these input parameters) is about 0.1 $\mu$m and is reached within one week. Both theoretical and empirical considerations agree that the grain formation time scale is very much shorter than the 1570 day pulsation period of OH 26.5+0.6.

Although the grain formation process is easily shown to take place on time scales short compared to the pulsation cycle, the question of grain destruction at the inner edge of the dust shell is much more complicated. A solid 0.1 $\mu$m radius sphere of quartz, for example, will sublimate only very slowly when raised 300 K (the rise in dust temperature at the inner shell boundary due to the increasing luminosity of the star) above the condensation temperature. It is very likely, however, that this rise in temperature will stop the dust formation process. This means that the minimum effect on the location of the inner dust shell during the rising portion of the star's light curve will be a motion outward at the outflow velocity of 14 km $s^{-1}$. This motion is nearly one-third the rate that the 1000 K dust temperature zone moves. If the grains are fragile composites of tiny grain pieces, with a small rise in grain temperature they may separate easily, leaving many small grains which then heat up to even higher temperatures and vaporize. These arguments suggest that while not instantaneous, the

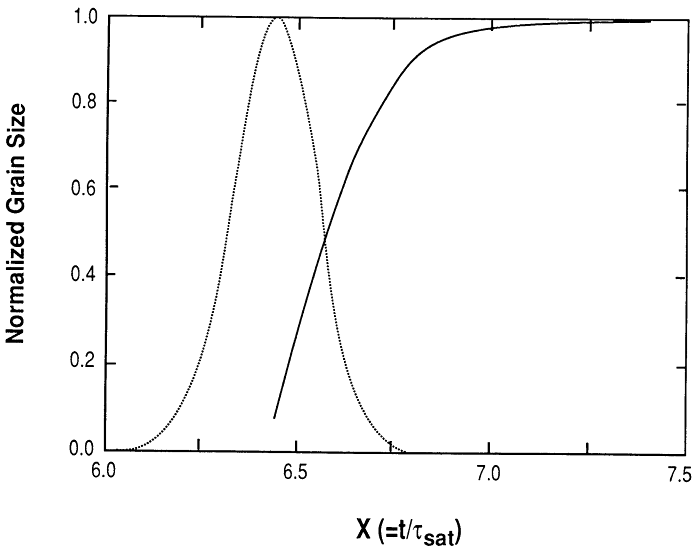


FIG. 2.—Theoretical grain growth curves based on the model of Yamamoto and Hasegawa (1977) applied to the conditions in the circumstellar shell of OH 26.5+0.6. The solid line is the ratio of the grain size to the final grain size as a function of time expressed in units of the saturation time. For the conditions in the circumstellar environment of OH 26.5+0.6, the saturation time is about 1 day. The dashed line is the volume number density of grain condensation nuclei formed per unit time (arbitrary vertical scale).

inner edge of the dust shell probably does not lag too far behind the 1000 K dust temperature zone during the rising portion of the pulsation cycle.

For simplicity, we will assume instantaneous grain formation/destruction at the inner edge of the dust shell. We recognize that grain destruction is probably not instantaneous, but the inner edge of the dust shell will move out at least half as fast as the condensation zone and perhaps faster. In the future, we plan to take a closer look at detailed changes in the shape of the spectrum of 26.5+0.6 with phase in order to investigate grain processes such as annealing (Nuth and Donn 1982) and a phase lag in the grain destruction using more accurate observations than presently available. The accuracy with which our model reproduces the observed changes in the spectral energy distribution of 26.5+0.6 suggests that grain destruction is taking place at some level during the rising portion of the light curve.

### III. PULSATION MODEL

Bowen (1988*a*, *b*) has developed a method for dynamical modeling of the atmospheres of long-period variable stars. The model atmosphere is driven by radial oscillations of its inner boundary (the "piston"), which simulate the effects of the pulsating stellar interior. The resulting behavior of the atmosphere is studied by numerically integrating the basic equations of hydrodynamics and thermodynamics, using standard finite difference methods. The equations include radiative transfer (using the Eddington approximation for a gray spherical atmosphere), radiation pressure on dust (but not a detailed treatment of grain formation and dynamics), and an approximate treatment of time-dependent thermal relaxation processes, including thermal interaction of gas and dust grains. The models invariably show shock waves; enormous extension of the dynamic atmosphere; complex velocity, temperature, and density structures; and a cool, low-speed wind. The wind develops primarily because of the shocks and the radiation pressure on dust, but the rate of mass loss is determined by interactions of several effects. Without special adjustment of any artificial fitting parameters, models calculated for stellar parameters like those of the Mira variables (periods of 200–500 days) have mass-loss rates in the same range as observed for Mira variables.

The dynamic models show increasing amounts of dust and increasing mass-loss rates as either the period is increased or the stellar mass is decreased. A larger period implies a larger radius, so either of these changes leads to lower surface gravity, a larger scale height, a more extended atmosphere, more dust, and more rapid mass loss. Low-mass models (around 1 $M_\odot$) with periods much greater than about 500 days typically have optically thick circumstellar dust and mass-loss rates of the order of or greater than $10^{-5}$ $M_\odot$ yr$^{-1}$. There is, in fact, a rapid transition in that region from shorter period models with properties like those of optical Mira variables to very long period, high-luminosity models which resemble the radio-luminous OH/IR stars. The latter models appear to offer a useful theoretical guide for characterizing the density distribution in the circumstellar dust shells of these stars.

One of us (G. H. B.) undertook the computation of dynamical atmosphere models with characteristics similar to those of OH 26.5+0.6. It proved relatively easy to do this for masses from about 1.0 to 1.4 $M_\odot$. Table 2 lists the input parameters and some calculated results for a 1.4 $M_\odot$ model with the observed luminosity, mass loss, and pulsation characteristics of 26.5+0.6. The parameters used for this model are reasonable in light of experience with many other dynamic models. The cross section for radiation pressure on dust, $k_{dust}$, was set to give a wind speed closely matching the observed value of 15 km s$^{-1}$; this corresponds to a reasonable gas-to-dust ratio, as discussed later in this paper. Best results were obtained by using an assumed mean dust condensation temperature, $T_c$, of 1100–1150 K, together with a relatively wide range of temperatures around that for the growth of grains. This value of $T_c$ is lower than is ordinarily used for the shorter period model but only slightly higher than is used in the dust shell model described in the next section. This condensation temperature may be more appropriate for stars like the RLOHIR stars, since grains may begin forming at higher temperatures from refractory silicates but continue to grow into dirty silicate grains by condensation of additional material at lower temperatures throughout an unusually large, dense circumstellar region. The piston velocity amplitude shown in Table 2 gave

TABLE 2

PULSATION MODEL PARAMETERS

| Input Parameters[a] | | Calculated Results | |
|---|---|---|---|
| $M$ | 1.4 $M_\odot$ | $\dot{m}$ | $6\times10^{-5}$ $M_\odot$ yr$^{-1}$ |
| $P$ | 1570 days | $V_{wind}$ | 14.9 km s$^{-1}$ |
| $R$ | 640 $R_\odot$[b] | $\tau_{dust}$ | 16.4[d] |
| $\langle T_{eff}\rangle$ | 2500 K[c] | $\langle L\rangle$ | 16,300 $L_\odot$[e] |
| $k_{dust}$ | 1.9 cm$^2$ g$^{-1}$ | $L_{max}/L_{min}$ | 2.94 |
| $T_c$ | 1100 K | | |

[a] Piston velocity amplitude = 2.0 km s$^{-1}$.
[b] At phase 0.25 or 0.75.
[c] $T_{eff}$ varies from 2375 to 2625 K.
[d] Average at phase 0.25 and 0.75.
[e] Time average over full cycle.

about the desired luminosity amplitude and mass-loss rate for this model, but its value is not critical.

Note that the radius and the effective temperature values used here differ from those for the dust shell model (2500 vs. 2200 K). They were arrived at as follows. The radius was set at the value (640 $R_{\odot}$) calculated from the period-mass-radius relation given by Ostlie and Cox (1986) for fundamental-mode pulsation of Mira variables. The luminosity and hence the effective temperature were then calculated from equation (4) in Iben (1984); $T_{\rm eff} = 2500$ K was about the lowest plausible value that could be obtained in this way, even using the rather low (Iben) mixing-length parameter, $1/H = 0.6$. Both of those equations have been used to calculate stellar parameters of dynamical models in other cases, and it seemed desirable to continue that use here. The differences between these values and those used in the dust shell model underscore the fact the two approaches are not entirely consistent, but they do not invalidate the use of the dynamical model in this paper. The dust shell model is only weakly dependent on the temperature and radius of the star (provided the total luminosity is consistent) but is strongly dependent on the radial density distribution and total optical depth of the shell.

Acceptable pulsation models of OH 26+0.6 probably could be constructed for a somewhat wider range of masses, though not without some difficulty and much care in adjusting the parameters. As the mass is increased, the radius required to give the observed period increases, and one must either accept a considerably greater luminosity than is believed correct, make rather extreme assumptions about the mixing length, or abandon entirely the use of the Iben relationship. It also becomes difficult to drive the model strongly enough to reach the observed mass-loss rate. For a smaller mass, there is little difficulty with the luminosity, but the model becomes so weakly bound gravitationally that even very low amplitude driving can give an excessive rate of mass loss.

It has not been our goal to insert a specific pulsation model into the dust code, but rather to explore the functional dependence on radius predicted by the pulsation model for density, velocity, and temperature, and to incorporate those results into a dust shell model. Thus we are not developing a truly self-consistent model in which the results of the dust code are fed back into the pulsation model and the modeling process iterated until a self-consistent set of physical parameters is obtained. That is not possible at this time because physical processes are treated at different levels of sophistication in the pulsation code and in the radiative transfer code. A fully self-consistent model remains a tantalizing project for the future.

Although an $r^{-2}$ density distribution is commonly used in dust shell models (Rowan-Robinson and Harris 1983; Jones, Hyland, and Robinson 1984), the exact form of the density distribution is usually considered a free parameter. Werner *et al.* (1980), for example, suggest that the density in the dust shell around 26.5+0.6 falls off more slowly than $r^{-2}$. We will not consider the form of the density distribution to be a free parameter but will determine it instead from the results of the pulsation models.

Figure 3 shows the dependence of the density and velocity on radius in the model described in Table 2. There is a region of relatively strong shocks but almost zero average velocity within about $3R_*$, then a region of rapid outward acceleration extending to perhaps $10R_*$, a little further acceleration out to about $20R_*$, and beyond that an almost steady outflow at constant velocity. Outside the acceleration region, the density

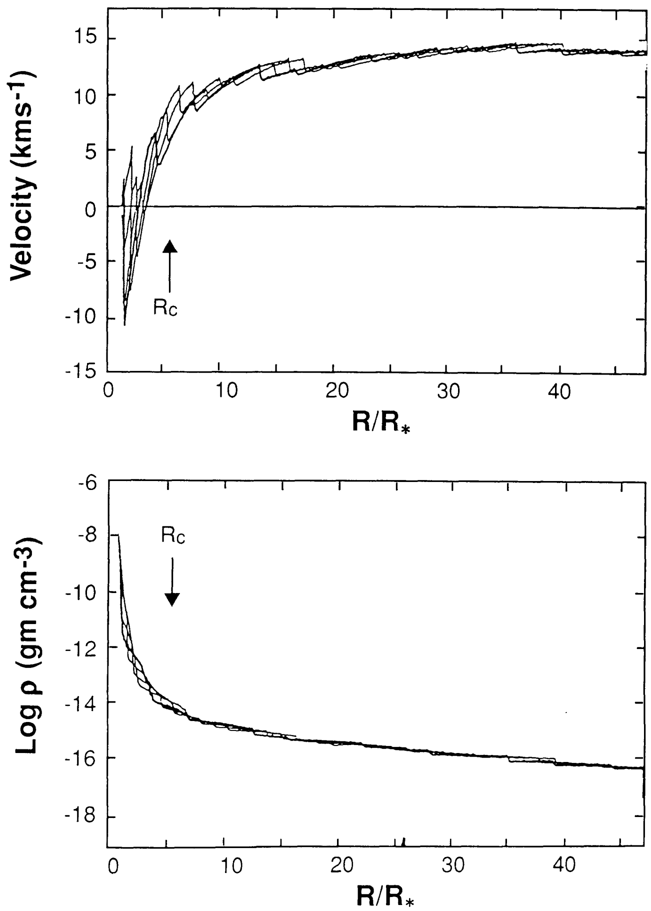


FIG. 3.—Radial density and velocity distributions for the pulsation model described in § III with input parameters given in Table 3. Curves are drawn for minimum, maximum, and phase 0.25.

follows an $r^{-2}$ distribution very closely, as in fact it must for steady outflow at almost constant speed. We have included three different phases in Figure 3 to show that a relatively smooth parameterization of the density distribution is possible.

To parameterize the density distribution, we will take the condensation radius, $R_c$, to be about $6R_*$ and use two power laws. For radii less than $3R_c$, where the grains are accelerated, we will use an $r^{-3}$ density distribution. Outside that radius we will use an $r^{-2}$ law, matching the two segments where they intersect. The actual value for the density at the condensation radius will not be taken from the pulsation mode but will be derived by fitting the dust model to the observations. Comparisons between the density distributions derived from the pulsation model and from the dust code model will be made in the next section.

## IV. DUST SHELL MODEL

Fix and Cobb (1987) find that the angular size of OH 26.5+0.6 is measurably larger at 10 $\mu$m than at 5 $\mu$m. This indicates that the silicate absorption at 9.7 $\mu$m is mostly circumstellar, with only a small interstellar contribution at most. The linear polarization of this star at 2.2 $\mu$m (Heckert and Zeilik 1981; our own, unpublished data) is about 2.5%. If this is purely interstellar, it suggests a minimum optical depth at 9.7 $\mu$m of about 0.3, and a maximum of less than 1.0 (Jones 1989) due to interstellar extinction alone. The effects of this extinction on the spectral energy distribution of OH 26.5+0.6 are relatively minor, and far less than the optical depth through the circumstellar shell. Also, any interstellar component will not be variable and will not contribute to the changes observed in the energy distribution of the star as a function of phase. As a consequence, we will assume that any interstellar extinction can be ignored in modeling this star.

In the previous sections we have constrained the optical properties of the dust, the size distribution of the dust grains, and the radial density distribution. Most remaining model constraints are provided by the observations of OH 26.5+0.6. These include the pulsation amplitude at 2.2, 3.5, and 5 $\mu$m (Engels *et al.* 1983; Jones *et al.* 1989; Kleinemann, Gillett, and Joyce 1982), low:resolution spectra at two phases in the light curve (Forrest *et al.* 1978) and an estimate of the minimum luminosity of the star (Table 1). This leaves only two parameters left to be specified: (1) the total optical depth of the dust shell at minimum phase and (2) the amplitude of the bolometric light curve. To model the dust shell around OH 26.5+0.6 we will use the code developed by Leung (1976) and very kindly provided by him for our use. In all models the dust shell is truncated at $1000R_c$.

First, the emergent energy distribution is modeled at minimum phase. At minimum phase, the only unspecified model parameter is the total optical depth of the shell. For a given $\tau_{9.7}$, the radiative transfer calculations are performed and the predicted spectrum compared with the observations. The value of $\tau_{9.7}$ is then adjusted until an acceptable fit is obtained. Once an acceptable fit is obtained, the dust density distribution of the shell is well determined and the luminosity of the central star can be increased.

Examination of Figure 3 shows that the actual density distribution as a function of phase is complex on scale lengths less than few $R_*$. Clearly the acceleration zone will more or less move in step with the condensation radius, so it is unphysical to simply take the density distribution at minimum phase and remove the inner portions as the star increases in luminosity and vaporizes the grains. As an approximate treatment we allow the acceleration zone (the $r^{-3}$ portion) to move out with the condensation radius, but keep the total optical depth through the shell what it would have been if material were simply removed from the inner edge in step with the change in luminosity. In this way, all of the grains that should be destroyed (or recondensed during the last half of the pulsation cycle) are destroyed but the presence of the acceleration zone is maintained.

The dust condensation radius increases as the increased luminosity of the star vaporizes the inner edge of the dust shell. This causes the total dust optical depth of the shell to decrease in a well-defined manner. A new emergent model spectrum results, and this can then be compared with the observations. The only relevant model parameter for this second part of the modeling process is the amplitude of the bolometric light curve of the star (assumed to be sinusoidal in shape). The larger the amplitude of the light curve, the greater the change in the dust optical depth, and the greater the change in the shape of the emergent spectrum.

A schematic representation of the model is shown in Figure 4. The emergent spectrum at three different phases in the pulsation cycle is compared with the observations in Figure 5. The basic parameters of the model are given in Table 3. Note that

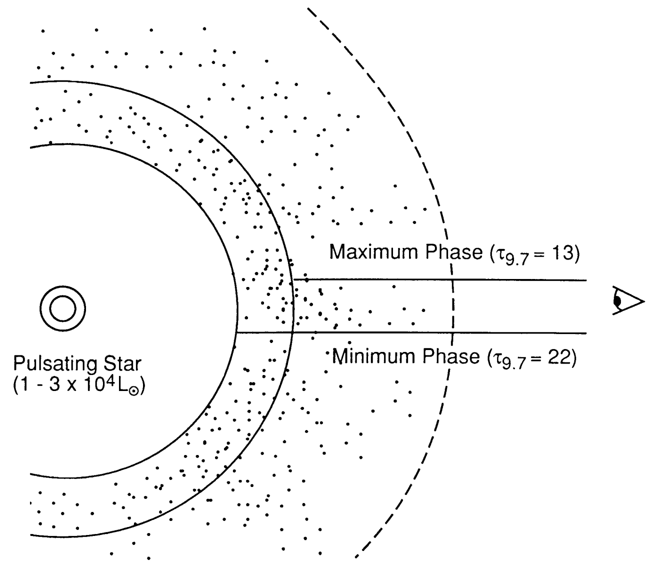


FIG. 4.—Schematic representation of the dust shell model

with the exception of the 20 $\mu$m feature, the entire spectrum from 1 to 13 $\mu$m is well fitted by the model at three phases in the pulsation cycle. Most importantly, the change in the depth of the silicate feature is accurately reproduced by the model. Also note that the large amplitude at the shorter wavelengths is reproduced, while at the same time the smaller amplitude seen in the continuum to either side of the 10 $\mu$m absorption band is correctly modeled. With our choice of the density law and grain properties, the changes in the spectrum of 26.5+0.6 can be accurately modeled by simply allowing the dust condensation radius to vary in step with the luminosity of the star.

A very interesting outcome of this modeling procedure is the surprisingly low amplitude of the bolometric light curve necessary to reproduce the observations. Based on $L$ (3.5 $\mu$m) and $M$

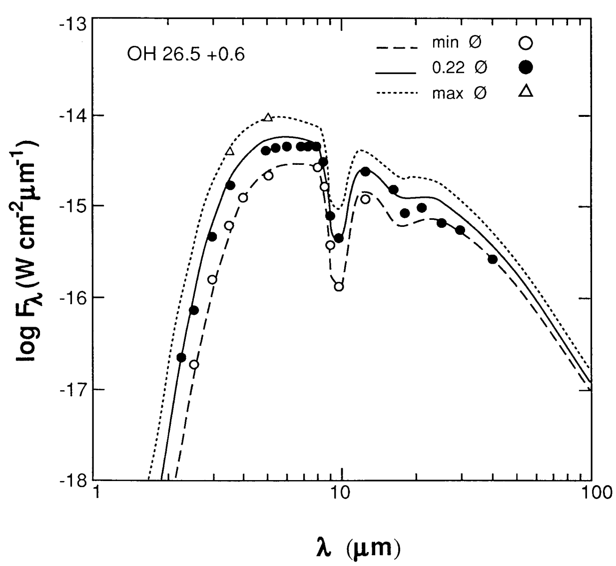


FIG. 5.—Comparison of the dust shell model's predicted energy distribution with the observed energy distribution at three different phases in the light curve.

TABLE 3

DUST MODEL PARAMETERS

| PARAMETER | PULSATION CYCLE PHASE | | |
|---|---|---|---|
| | Minimum | 0.25 | Maximum |
| Luminosity ($L_\odot$) ………… | $1.0 \times 10^4$ | $1.8 \times 10^4$ | $3.0 \times 10^4$ |
| Temperature (K) ……….. | 2000 | 2200 | 2400 |
| Radius ($R_\odot$) ……………. | 846 | 938 | 1017 |
| $T_c$ (K) …………………… | 1000 | 1000 | 1000 |
| $R_c$ ($R_\odot$) ………………… | 5100 | 6650 | 8645 |
| $\tau_{9.7}$ …………………….. | 23 | 16 | 13 |

(5 μm) photometry and bolometric corrections derived from the $L-M$ color, Jones (1987) suggests that the swing in total luminosity for 26.5+0.6 is a factor of 8! Our modeling predicts a much more sedate factor of 3 swing in total luminosity. The significant increase in the total dust optical depth at minimum phase compared to maximum, even though it is taking place in the inner, hotter region of the dust shell, strongly effects the amount of emergent 1–4 μm light, causing the near-infrared light curve to have a large amplitude. It also, of course, deepens the silicate absorption feature compared to the depth at maximum light.

As the star changes in luminosity, not only does the inner edge of the dust shell move radially, but the temperature distribution of the dust moves radially as well. As the star decreases in brightness from maximum, the condensation radius moves inward, adding to the total column density of dust, and *lowering* the temperature of the dust everywhere else in the shell. This reduces the source function in the outer layers of the dust shell, reducing the emergent flux at wavelengths where the opacity is high. The model dust temperature distribution is shown in Figure 6. We find it encouraging that incorporation of such a simple process into the model can so well reproduce the emergent spectrum over a decade in wavelength at the three phases of the pulsation cycle.

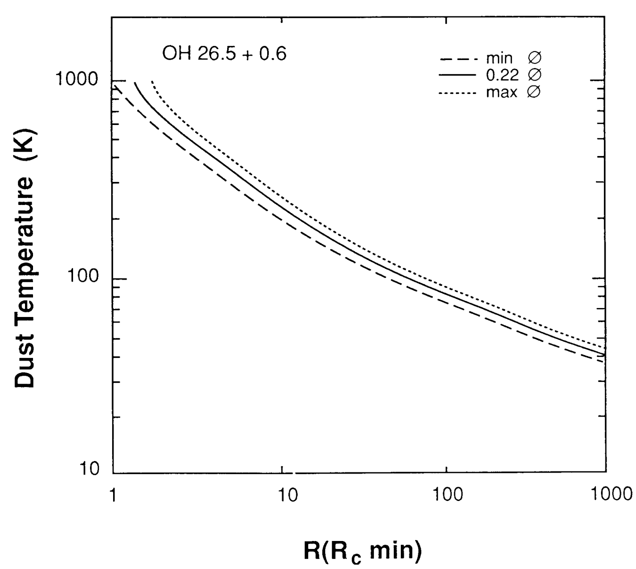


FIG. 6.—Radial dependence of the model dust temperature as a function of phase.

Longward of 20 μm, the model predicts about 30% more flux than is observed, in spite of the increased opacity in our grains at those wavelengths. A further increase in the opacity of the grains at long wavelengths would cause the model to sample the source function further out from the star where the temperature is lower, resulting in less flux. Note that the dust temperature drops below 200 K by $10R_*$, well before optical depth unity to the outer edge of the shell is reached at 20 μm. This suggests that an improved fit can be obtained if the temperature dependence of the opacity was explicitly taken into account, rather than setting the dust absorption coefficients the same everywhere in the shell.

We have demonstrated that fixing the form of the density distribution based on pulsation models will result in accurate results for a phase-dependent dust shell model. Other forms of the density distribution, including a simple $r^{-2}$ law, are also capable of producing good model fits. We have shown that a density distribution based on a pulsation model is consistent with the demands of the dust shell model and need no longer be considered a free parameter. The actual values for the density and temperature as a function of radius were computed separately for the pulsation model and the dust shell model. The two models agree fairly well in the location of the dust condensation radius and agree well in the temperature distribution in the outer portions of the shell. A direct comparison of the density between the radiative transfer and pulsation models is complicated by the fact that the pulsation model uses a single mean dust opacity of 1.9 $cm^2$ per gram of total material, whereas the radiative transfer model incorporates the wavelength dependence of the opacity explicitly but deals only in terms of optical depth. At phase 0.25, the radiative transfer model requires a dust density at the condensation radius of $1 \times 10^{-17}$ g $cm^{-3}$ assuming a density for the grains of 3.3 gm $cm^{-3}$ (Suh 1988). The gas density at the corresponding radius in the pulsation model (Figure 3) is $3 \times 10^{-15}$ gm $cm^{-3}$, yielding a gas to dust ratio of about 300. The dust opacity used by Bowen in the pulsation model corresponds to a gas to dust ratio of about 500 for a 1000 K radiation field (a rough average for the inner shell).

The agreement in density between the two models is remarkable considering the very different physical phenomena being modeled. This result is encouraging and suggests the two models can be combined in a self-consistent manner. In the future we plan to explore techniques for incorporating the results of the two models together by feeding back the results of one into the other until a self-consistent set of physical parameters at all radii is obtained.

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