Chapter 2 Binary Image Processing

continuous gray level -> quantized 256, 4092
binary image
    less expensive hardware
    fast processing

in case silhouette is enough
    special illumination
    only few objects in a scene

object pixel ; 1 -> black
background pixel ; 0 -> white

- formation of binary images
- geometric properties
- topological properties
- object recognition

2.1 Thresholding

problem; identify the subimages of object
    very difficult problem for computers

segmentation; partitioning of an image into regions
    regions; an object candidate

Def. 2.1 A region is a subset of an image
Def. 2.2 Segmentation is grouping pixels into regions s.t.

- $\bigcup_{i=1}^{k} P_i = \text{entire image}$ ($\{P_i\}$ is an exhaustive partitioning)
- $P_i \cap P_j = 0, \quad i \neq j$ ($\{P_i\}$ is an exclusive partitioning)
- Each region P_i satisfies a predicate; all points of the partition have some common property.
  has uniform intensity
- Pixels belonging to adjacent regions, when taken jointly, do not satisfy the predicate.

Thresholding

object-background separation

$$B[i,j] = F_T[i,j]$$

for a darker object on a lighter background

$$F_T[i,j] = \begin{cases} 1 & \text{if } F_T[i,j] \le T \\ 0 & \text{otherwise} \end{cases}$$

if object intensity is in a range

$$F_T[i,j] = \begin{cases} 1 & \text{if } T_1 \le F_T[i,j] \le T_2 \\ 0 & \text{otherwise} \end{cases}$$

if several disjoint intervals is,

$$F_T[i,j] = \begin{cases} 1 & \text{if } F[i,j] \in Z \\ 0 & \text{otherwise} \end{cases}$$

Fig. 22

Comment)

selection of threshold; experience; case by case

automatic thresholding

2.2 Geometric Properties

assumptions; camera location and environment are known

objects are different in size and shape

only one object

2.2.1 Size

area; zeroth order moment

$$A = \sum_{i=1}^{n} \sum_{j=1}^{m} B[i,j]$$

2.2.2 Position

position of object is important

known; position of camera relative to the table

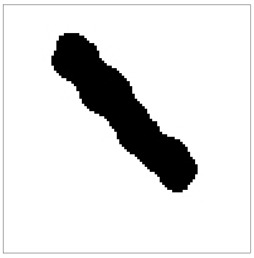
enclosing rectangle

center of area

$$\bar{x} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} j\,B[i,j]}{A}$$

$$\bar{y} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} i\,B[i,j]}{A}$$

2.2.3 Orientation

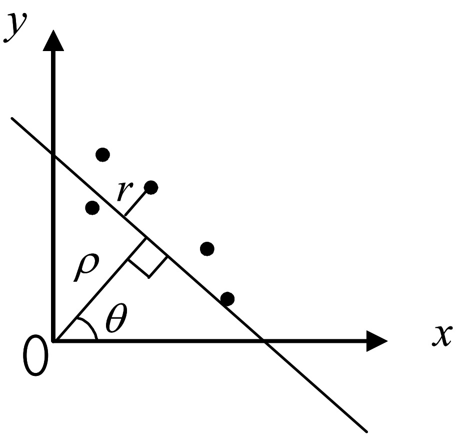


elongated object -> orientation

find a line s.t

minimize $\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{m} r_{ij}^2 B[i,j]$

r_{ij} : 점 (i,j)로부터 직선에 수직인 거리



극좌표 직선식; $\rho = x\cos\theta + y\sin\theta$

* 모든 점으로부터 거리의 합을 최소화하는 직선 찾기

- for ρ

  for a point (x,y)

  $r^2 = (x\cos\theta + y\sin\theta - \rho)^2$

  $\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{m} (x_{ij}\cos\theta + y_{ij}\sin\theta - \rho)^2 B[i,j]$

  let $\frac{\partial \chi^2}{\partial \rho} = 0$

  => $\rho = \bar{x}\cos\theta + \bar{y}\sin\theta$

Comment) pass through center point

- for θ

  $x' = x - \bar{x}$, $y' = y - \bar{y}$

  $\chi^2 = a\cos^2\theta + b\sin\theta\cos\theta + c\sin^2\theta$

$$a = \sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij}^{'})^2 B[i,j]$$
$$b = 2\sum_{i=1}^{n}\sum_{j=1}^{m}x_{ij}^{'}y_{ij}^{'}B[i,j]$$
$$c = \sum_{i=1}^{n}\sum_{j=1}^{m}(y_{ij}^{'})^2 B[i,j]$$

let $\frac{\partial\chi^2}{\partial\theta} = 0$

=> $\tan(2\theta) = \frac{b}{a-c}$

$$\sin(2\theta) = \pm\frac{b}{\sqrt{b^2+(a-c)^2}}$$
$$\cos(2\theta) = \pm\frac{a-c}{\sqrt{b^2+(a-c)^2}}$$

if b=0, a=c -> no unique axis

Elongation $E = \frac{\chi_{max}}{\chi_{min}} \geqq 1$

Comment) atan2를 이용, elongation is 1 for circle

2.3 Projections

finding no. of 1 pixels on lines to each bins

- Horizontal & Vertical projection

$$H[i] = \sum_{j=1}^{m} B[i,j]$$
$$V[j] = \sum_{i=1}^{n} B[i,j]$$

Area, Center

$$A = \sum_{j=1}^{m} V[j] = \sum_{i=1}^{n} H[i]$$
$$\bar{y} = \frac{\sum_{i=1}^{n} iH[i]}{A}$$
$$\bar{x} = \frac{\sum_{j=1}^{m} jV[i]}{A}$$

- Diagonal projection

compute index for the histogram bucket for the current (i,j)

- affine transform (linear combination with constant)

$$d = ai + bj + c$$

- pixel at upper right (0, m-1) into first
  lower left (n-1, 0) into last

No. of bucket; n+m-1 i.e. (0 -> n+m-2)

$$a \cdot 0 + b(m-1) + c = 0$$
$$a(n-1) + b \cdot 0 + c = n + m - 2$$
$$a = -b$$

=> $d = i - j + m - 1$

Comment) Projection; useful features for recognition
compact representation, fast algorithm

2.4 Run-Length Encoding

compact representation of a binary image
length of the runs of 1 pixels

(1) start position, length of runs of 1s

(2) only length of runs starting with that of 1 runs

using (2)

$r_{i,k}$; length of the kth run in the ith row

$$A = \sum_{i=0}^{n-1} \sum_{k=0}^{(\frac{m_i-1}{2})} r_{i,2k+1}$$

m_i; number of runs in the ith row

2.5 Binary Algorithms

grouping object pixels together
spatially close; spatial proximity

2.5.1 Definitions

Neighbors

p[i,j] has 4 pixels with common boundary
4 pixels with common corner

* Two pixels are 4-neighbors; if two pixels are with common boundary
* 8-neighbor; if two pixels are with common corner

p[i,j] has 4 neighbors [i+1,j], [i-1,j], [i,j+1], [i,j-1] -> 4 connected
8 neighbor; 4 neighbor + [i+1, j+1], [i+1, j-1], [i-1,j+1], [i-1,j-1]
-> 8 connected

Path

$[i_0, j_0] \rightarrow [i_n, j_n]$

path; $[i_0, j_0], [i_1, j_1], [i_2, j_2] \cdots [i_n, j_n]$
$[i_k, j_k]$, $0 \le k \le n-1$; 4 path when using 4-connected
8 path when using 8-connected

Forground

set of 1 pixels ; S

Connectivity

- $p \in S$ is said to be connected to $q \in S$ if there is a path from p to q consisting entirely of pixels of S
- equivalance relation

  for $p, q, r \in S$

  (1) reflexivity; p is connected to p

  (2) commutatively

  if p is connected to q , then q is connected to p

  (3) transitivity

  if p is connected to q and q is connected to r, then p is connected to r

Connected components; object

A set of pixels in which each pixel is connected to all other pixels is called a connected component

Background

Background; set of all connected components of $\overline{S}$ that have points on the image border

Holes; all other connected components $\overline{S}$ except backgrond

for S (object); 8 connectedness

for $\overline{S}$ (hole, background); 4 connectedness

Boundary(edge)

S'; boundary of S is the set of pixels of S that have 4 neighbor in $\overline{S}$

Interior

interior of S is (S-S′)

Sourrounds

S is inside T

if any 4 path from any point of S to the border of the pixture

must intersect T

2.5.2 Component Labeling

major task; finding connected components in an image
surface <=> spatially close
one object in an image
many object in an image

Recursive Algorithm

Algorithm 2.1
(used on parallel processor)

(1) Scan the image to find an unlabeled 1 pixel and assign it a new label L
(2) Recursively assign a label L to all its 8 neighbor
(3) Stop if there are no more unlabeled 1 pixels
(4) goto step (1)

Sequential Algorithm

Algorithm 2.2
(using 4 connectivity)

(1) Scan the image left to right, top to bottom
(2) if the pixel is 1, then

(a) If only one of its upper and left neighbors has a label, then
copy the label
(b) If both have the same label, then copy the label
(c) If both have different labels, then
copy the upper label
and enter the labels in the equivalence table as equivalent labels
(d) Otherwise assign a new label to this pixel
and enter this label in the equivalence table
(3) If there is more pixels to consider, then goto step (2)
(4) Find the lowest label for each equivalent set in the equivalence table
(5) Scan the pixture, Replace each label by the lowest label in its equivalent set

2.5.3 Size flter

due to noise

object; no. of pixels >= T_0

noise; no. of pixels < T_0

2.5.4 Euler number

feature of an object

topological feature; invariant to translation, rotation, scaling

$$E = C - H$$

C; number of connected components

H; number of Holes

2.5.5 Region boundary

Algorithm 2.3 Boundary following Algorithm

(1) Find the starting pixel $s \in S$ for the region using a systematic scan, i.e. from left to right and from top to bottom of the image

(2) current pixel on boundary c=s, let 4 neighbor to the west of s be $b \in \overline{S}$

(3) 8-neighbor of c starting with b in clockwise order; n_1, n_2, ... n_8

Find n_i for the first i that is in S

(4) set $c=n_i$, and $b=n_{i-1}$

(5) Repeat step (3) and (4) until c=s

2.5.6 Area and Perimeter

Area; number of pixels in S
along with labeling

Perimeter; definitions

(1) sum of lengths of the cracks; line between $p \in S$ and $q \in \overline{S}$
(2) number of steps taken by a boundary following algorithm
(3) number of boundary pixels

2.5.7 Compactness

isoperimetric inequality

$\frac{P^2}{A} \geq 4\pi$ P; perimeter, A; Area

circle; most compact

for a line; compact is ∞
compactness of square < one of rectangle

2.5.8 Distance Measure

distance b.t.w. two pixels

conditions of for distance measure

$d(p,q) \geq 0$ and $d(p,q) = 0$ iff $p = q$
$d(p,q) = d(q,p)$
$d(p,r) \leq d(p,q) + d(q,r)$

Euclidean;

$d_{Euclidean}([i_1, j_2],[i_2, j_2]) = \sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2}$; computationally intensive, real value
=> integer valued square

City-block;
$d_{city} = |i_1 - i_2| + |j_1 - j_2|$

Chessboard;
$d_{chess} = \max(|i_1 - i_2|, |j_1 - j_2|)$

disc of k ; set of pixels at distance <= k

2.5.9 Distance transforms

Distance transform; obtaining an image representing distance from $p \in S$ to $q \in \overline{S}$

$$f^0[i,j] = f[i,j]$$
$$f^m[i,j] = f^0[i,j] + \min(f^{m-1}[u,v])$$

for all [u,v]; four neighbor of [i,j] s.t. $d([u,v],[i,j]) = 1$

m; iteration number

2.5.10 Medial Axis

distance is locally maximum if

$$d([i,j],\overline{S}) \geq d([u,v],\overline{S})$$

for all [u,v]; four neighbor of [i,j] s.t. $d([u,v],[i,j]) = 1$

set of pixels which are locally maximum; S*

skeleton, symmetric axis, medial axis

inner pixel detection; using medial axis and maximal disc at each pixel

2.5.11 Thinning

binary valued image regions -> line; approximately center line, skeletons , core line
- reduce image components to their essential informations
- for elongated shapes
- document analysis, character stroke

Thinning requirements;
(1) Connected image regions must thin to connected line structure; connectivity
(2) Thinned result should be minimally 8 connected; minimal number of pixels
(3) Approximate endline locations should be maintained; not make short
(4) Thinning results should be approximate the medial lines
(5) Extraneous spurs (short branches) caused by thinning should be minimized.

Iteratively peeled off
window of nxn pixles
peeled off one boundary layer

2.5.12 Expanding and Shrinking

Expanding; change a pixel from 0 to 1 if any neighbors of the pixel are 1
Shrinking; chnage a pixel from 1 to 0 if any neighbor of the pixel are 0
expanding background

$$(S^m)^{-n} \neq (S^{-n})^m$$
$$(S^{(m-n)})$$

$$S \subset (S^k)^{-k}$$
$$S \supset (S^{-k})^k$$

General case; dilation
erosion

2.5.6 Morphological Operators

from the study of shape
operation between two binary image A, B

Intersection;

$A \cap B = \{p | p \in A \text{ and } p \in B\}$

Union;

$A \cup B = \{p | p \in A \text{ or } p \in B\}$

Complement;

$\overline{A} = \{p | p \in \Omega \text{ and } p \notin A\}$, Ω; all pixels are 1

vector sum of p[i,j] and q[k,l] => p+q at [i+k, j+l]
vector difference => at [i−k, j−l]