Ch. 9 Control system design by frequency response

9.1 Introduction
transient response performance is important

After open loop design by frequency response method
closed loop poles/zeros are determined
if not satisfactory, compensator designed

experimentally obtained frequency response for system components easily combined.

Two approach;
polar plot
Bode diagram; more simpler
(i) adjust open loop gain -> to meet steady state accuracy
(ii) draw the mag. / phase curve of the uncompensated open loop
(iii) check gain/phase margin; if not satisfactory, design a suitable compensator
(iv) check any other requirements

Information obtainable from open-loop frequency response
low freq. region; steady state behavior of the c.l. system
medium(-1+j0 near) freq. region; relative stability
high freq. region; complexity of the system

Requirements on open-loop freq. response
compensation;
compromise b.w. steady state accuracy & relative stability

low freq. region; gain -> high
near gain crossover freq. ; slope is -20dB/decade
high freq. region; attenuate as rapidly as possible

Basic characteristics of lead, lag, and lag-lead compensation;
lead compensation; transient response improvement
(order 1 increase) small change in steady state accuracy
accentuate high-freq. noise effects
lag compensation; steady state accuracy improvement
(order 1 increase) increase transient response
suppress high freq. noise effects
lag-lead; combines both characteristics, order 2 increase

9.2 Lead compensation

Characteristic of lead compensation

$$K_c\alpha\frac{Ts+1}{\alpha Ts+1}=K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}},\qquad (0<\alpha<1)$$

min(α) = 0.05 -> maximum phase angle = 65°

$$\sin\phi_m=\frac{\frac{1-\alpha}{2}}{\frac{1+\alpha}{2}}=\frac{1-\alpha}{1+\alpha}\qquad (9.1)$$

$$\log\omega_m=\frac{1}{2}\left(\log\frac{1}{T}+\log\frac{1}{\alpha T}\right)$$

=> $\omega_m=\frac{1}{\sqrt{\alpha}\,T}$

* lead compensator is high pass filter

Lead compensation techniques based on the frequency response approach

provide sufficient phase lead angle to the phase lag system

Design procedure

(1) determine gain K to obtain static error constatnt

$$G_c(s)=K_c\alpha\frac{Ts+1}{\alpha Ts+1}=K\frac{Ts+1}{\alpha Ts+1},\qquad (0<\alpha<1)$$

open loop T.F.

$$G_c(s)G(s)=K\frac{Ts+1}{\alpha Ts+1}G(s)=\frac{Ts+1}{\alpha Ts+1}G_1(s),\qquad (0<\alpha<1)$$

(2) draw Bode diagram of $G_1(j\omega)$, evaluate Phase margin

(3) determine necessary phase lead angle ?

(4) determine α using eq. (9.1)

determine freq. at $|G_1(j\omega)|=-20\log(\frac{1}{\sqrt{\alpha}})$ => new gain crossover freq.

$$\omega_m=\frac{1}{\sqrt{\alpha}\,T},\ \phi_m$$

(5) determine the corner freq. of the lead compensator

zero; $\omega=\frac{1}{T}$, pole; $\omega=\frac{1}{\alpha T}$

(6) determine

$$K_c=\frac{K}{\alpha}$$

(7) check the gain margin

Ex. 9.1)

9.3 Lag compensation

Characteristic of lag compensator

$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}, \qquad (\beta > 1)$$

zero; $s = -\frac{1}{T}$, pole; $s = -\frac{1}{\beta T}$

* lag compensator is low pass filter

Lag compensation techniques based on the freq. response approach

1. assume

$$G_c(s) = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = K\frac{Ts+1}{\beta Ts+1}, \qquad (\beta > 1)$$

open loop T.F.

$$G_c(s)G(s) = K\frac{Ts+1}{\beta Ts+1}G(s) = \frac{Ts+1}{\beta Ts+1}G_1(s), \qquad (\beta > 1)$$

determine gain K to obtain static error constant

2. find the freq. of the open loop T.F. at
   phase angle = −180 + specified phase margin + (5^o ~ 12^o)
   -> new gain crossover freq.

3. choose the corner freq. $\omega = \frac{1}{T}$ 1 octave to 1 decade
   below the new gain crossover freq.

4. at the new gain crossover freq.
   attenuation = $-20\log\beta$, => determine the value of β
   the other corner freq. (from the pole of the lag compensator) ; $\omega = \frac{1}{\beta T}$

5. determine $K_c = \frac{K}{\beta}$

9.4 Lag-Lead compensation