Ch. 8 Frequency response analysis

8-1 Introduction

steady state response of a system to a sinusoidal input
Nyquist stability condition

* steady state output to sinusoidal input

in Chapter 5;

$$x(t) = X\sin\omega t$$

$$Y(s) = G(s)X(s) = G(s)\frac{\omega X}{s^2+\omega^2}$$
$$= \frac{a}{s+j\omega} + \frac{\overline{a}}{s-j\omega} + \frac{b_1}{s+s_1} + \cdots + \frac{b_n}{s+s_n}$$

Let $G(j\omega) = |G(j\omega)|e^{j\phi}$

$$y_{ss}(t) = X|G(j\omega)|\frac{e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}}{2j}$$
$$= X|G(j\omega)|\sin(\omega t+\phi)$$
$$= Y\sin(\omega t+\phi)$$

Phase shift $\phi = \tan^{-1}\left[\frac{\text{imaginary part of } G(j\omega)}{\text{real part of } G(j\omega)}\right]$

- Bode diagram or logarithmic plot
- Nyquist plot or polar plot
- Log-magnitude v.s. phase plot

8-2 Bode diagrams

multiplication => addition
asymptotic approximation

Decibel; $20\log|G(jw)|$

Basic factors of $G(jw)H(jw)$

- Gain; K
- Integral and derivative factor; $(jw)^{\mp}$
- 1st order factor; $(1+jwT)^{\mp}$
- 2nd order factors; $(1+2\zeta(jw/w_n)+(jw/w_n)^2]^{\mp}$

- Gain K

$$20\log(K\text{x}10^n) = 20\log K + 20n$$

Phase angle; 0

- Integral and derivative factor; $(jw)^{\mp}$

$20\log|\frac{1}{jw}| = -20\log w$ dB, Phase angle; -90^o

$20\log|jw| = 20\log w$ dB, Phase angle; 90^o

$20\log|\frac{1}{(jw)^n}| = -20n\log w$ dB. Phase angle; -90^oxn

$20\log|(jw)^n| = 20n\log w$ dB, Phase angle; 90^oxn

- 1st order factor; $(1+jwT)^{\mp}$

$$20\log|\frac{1}{1+jwT}| = -20\log\sqrt{1+w^2T^2}\ \text{dB}$$

if w << 1/T

$$-20\log\sqrt{1+w^2T^2} \simeq -20\log 1 = 0\ \text{dB}$$

if w>> 1/T

$$-20\log\sqrt{1+w^2T^2} \simeq -20\log wT\ \text{dB}$$

Phase angle

$/\frac{1}{1+jwT} = -\tan^{-1}wT$; at w = 1/T, ϕ = 45^o

$$/1+jwT = \tan^{-1}wT = -/\frac{1}{1+jwT}$$

- 2nd order factors; $(1+2\zeta(jw/w_n)+(jw/w_n)^2]^{\mp}$

$$20\log\left|\frac{1}{1+2\zeta(j\frac{w}{w_n})+(j\frac{w}{w_n})^2}\right| = -20\log\sqrt{(1-\frac{w^2}{w_n^2})^2+(2\zeta\frac{w}{w_n})^2}$$

if $w \ll w_n$ mag = 0 dB

$w \gg w_n$ mag = $-20\log\frac{w^2}{w_n^2} = -40\log\frac{w}{w_n}$ dB

Phase angle;

$$\phi = \angle\frac{1}{1+2\zeta(j\frac{w}{w_n})+(j\frac{w}{w_n})^2} = -\tan^{-1}\left[\frac{2\zeta\frac{w}{w_n}}{1-(\frac{w}{w_n})^2}\right]$$

if $w \ll w_n$ ϕ = 0°

$w = w_n$ ϕ = 90°

$w \gg w_n$ ϕ = -180°

Minimum-phase system and Nonminimum phase system

- Minimum phase T.F; pole or zero on left half plane => minimum phase system
- Nonminimum phase T.F.; 〃 on right half plane => nonminimum phase system

Example;

$$G_1(jw) = \frac{1+jwT}{1+jwT_1}, \quad G_2(jw) = \frac{1-jwT}{1+jwT_1}, \quad 0 < T < T_1$$

since $G_2(jw) = G_1(jw)\frac{1-jwT}{1+jwT}$

magnitude of G is always 1, mag. of G_1 = mag. of G_2

phase angle of $\frac{1-jwT}{1+jwT} = -2\tan^{-1}wT$ => 0 ~ 180°

For a Minimum phase sys. ; the mag. and phase angle characteristic is uniquely related.

phase angle at $w=\infty$, $-90^o(q-p)$ <= s^p/s^q

magnitude at 〃 , $-20(q-p)$ dB/decade

For a Nonminimum phase sys.; 〃 not 〃

phase angle at $w=\infty$ is not $-90^o(q-p)$ <= s^p/s^q

magnitude at 〃 , $-20(q-p)$ dB/decade

Transport Lag.

$$G(jw) = e^{-jwT}$$

$$|G(jw)| = |\cos wT - j\sin wT| = 1$$

$$\angle G(jw) = -wT\ (rad) = -57.3\,wT\ (\deg.)$$

Example 8.2

$$G(jw) = \frac{e^{-jwL}}{1 + jwT}$$

8-4 Polar plots

Integral and derivative factors; $jw, \quad \frac{1}{jw}$

First order factors; $1 + jwT, \quad \frac{1}{1 + jwT}$

Quadratic factors;

$$[1 + 2\zeta \frac{jw}{w_n} + (\frac{jw}{w_n})^2]^{-1} \quad ; \quad \text{for } \zeta > 0$$

$$\lim_{w \to 0} G(jw) = 1/0^o, \quad \lim_{w \to \infty} G(jw) = 0/-180^o \quad ,$$

$$\frac{1}{[1-(w/w_n)^2] + 2\zeta(jw/w_n)}$$

$$= \frac{[1-(w/w_n)^2] - 2\zeta(jw/w_n)}{[1-(w/w_n)^2]^2 + 4\zeta^2(w/w_n)^2} \simeq \frac{-w/w_n - 2\zeta j}{(w/w_n)^3 + 4\zeta^2(w/w_n)}$$

$$[1 + 2\zeta \frac{jw}{w_n} + (\frac{jw}{w_n})^2], \quad \text{for } \zeta > 0$$

$$\lim_{w \to 0} G(jw) = 1/0^o, \quad \lim_{w \to \infty} G(jw) = \infty/180^o$$

8-7 Nyquist stability criterion

Relates freq. response of $G(jw)H(jw)$ to the no. of poles and zeros of 1+GH in LFP

for stable;

all roots of $1+G(s)H(s)=0$; left half s-plane

* Nyquist stability criterion

(no poles or no zeros on jw axis)

$\lim_{s\to\infty} G(s)H(s) = \text{const.}$

if the G(s)H(s) has k poles in right half s plane,

for stability, the locus encircles the −1+ j0 point k times in the counter clockwise direction.

- Remarks

(1) Z= N+P

Z; no. of zeros of 1 +G(s)H(s) in the right half plane

N; no. of clockwise encirclements of −1+j0 point

P; no. of poles of G(s)H(s) in the right half plane

(2) unstable inner loop => apply Routh's stability criterion

Transport lag e^{-Ts}

$$e^{-Ts} \simeq \frac{1-\frac{Ts}{2}}{1+\frac{Ts}{2}}$$

(3) if locus of $G(jw)H(jw)$ pass through −1+j0 point,

then closed loop poles are located on the jw axis

8-8 Stability analysis

If Nyquist path in s-plane encircles Z zeros and P poles of 1+G(s)H(s) and doesn't pass through any poles or zeros of 1+G(s)H(s) as s moves clockwise along the Nyquist path,

contour in G(s)H(s) plane encircles the −1+j0 point N=Z−P times in clockwise direction.

* Conditionally stable system

8-9 Relative stability

Phase margin and Gain margin

for minimum-phase system,

both the phase margin and gain margin must be positive.

negative gain margin => unstable