Ch. 7 Control systems design by the Root-Locus Method

7.1 Introduction

Performance:

System compensation:

gain adjustments; 1st step
increase steady state accuracy
poor stability, instability
redesign ; modifying the structure
additional devices or components <- compensator

series compensation;
feedback (parallel) compensation;
choice; depends on the nature of the signal
device available
economic considerations

Depends on phase angle
lead network(compensator)
lag network(compensator)
lead-lag network(compensator); lag in low frequency , lead in high frequency

7.2 Preliminary design consideration

Effect of adding poles to the open loop T.F.
pulling the root locus to the right

Effect of the addition of zeros to the open loop T.F.
pulling the root locus to the left

7.3 Lead compensation

speeds up the response
increase the stability

using OP amp
RC circuit
Spring-dashpot

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2R_4}{R_1R_3}\frac{R_1C_1s+1}{R_2C_2s+1} = \frac{R_4C_1}{R_3C_2}\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}$$

$$= K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

$$\text{where,} \ T = R_1C_1, \quad \alpha T = R_2C_2, \quad K_c = \frac{R_4C_1}{R_3C_2}$$

DC gain; $K_c\alpha = \frac{R_2R_4}{R_1R_3}$

if $R_1C_1 > R_2C_2$ or $\alpha < 1$; lead network
if $R_1C_1 < R_2C_2$; lag network

* Lead compensation technique based on the root=locus approach

(1) determine the desired location for the dominant closed loop poles
(2) draw root locus
ascertain whether gain adjustment fulfills
if not, how much angle needs
(3) G_c ; determine α, T from needs angle, K_c from open loop gain
(4) α ; as large as possible (larger velocity constant K_v)
(5) determine the open loop gain from the magnitude cond.

(Ex. 7-1)

$$G(s) = \frac{4}{s(s+2)}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2+2s+4} = \frac{4}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}$$

damping ratio or closed loop; 0.5
closed loop $\omega_n = 2$ rad/sec.
$K_v = 2\,\mathrm{sec}^{-1}$

Desired closed loop poles; $s = -2 \pm j2\sqrt{3}$

Procedure;

Total sum of the angles = $\pm 180^o(2k+1)$

$$G_c(s)G(s) = \left(K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \right) G(s), \qquad (0 < \alpha < 1)$$

Angle of the present system at the desired poles

$$\angle \frac{4}{s(s+2)} \bigg|_{s=-2+j2\sqrt{3}} = -210^o$$

the lead compensator must add $\phi = 30^o$
=> zeros at s =-2.9, poles at s=-5.4

$$\left(T = \frac{1}{2.9} = 0.345, \qquad \alpha T = \frac{1}{5.4} = 0.185 \right)$$

=>

$$G_c(s)G(s) = \left(K_c \frac{s+2.9}{s+5.4} \right) \frac{4}{s(s+2)}$$

Magnitude cond.

$$\left| \frac{K(s+2.9)}{s(s+2)(s+5.4)} \right|_{s=-2+j2\sqrt{3}} = 1$$

=> K= 18.7 => $K_c = 18.7/4 = 4.68$

=> $G_c(s) = 2.51 \dfrac{0.345s+1}{0.185s+1} = 4.68 \dfrac{s+2.9}{s+5.4}$

determine $R_1, C_1, R_2, C_2, R_3, R_4$

7.4 Lag compensation

When
system shows satisfactory transient response
unsatisfactory steady state characteristics

So, dominant poles should not be changed significantly
but open loop gain should be increased

Angle added by lag network should be $< 5^o$ (small)
Place pole and zero of $G_c(s)$ **closely**, and **near origin** of s-plane ;

Original system ; $K_v = \lim_{s \to 0} s\, G(s)$
When lag compensator added;

$$\begin{aligned} \widehat{K_v} &= \lim_{s \to 0} s\, G_c(s) G(s) \\ &= \lim_{s \to 0} G_c(s) K_v \\ &= \widehat{K_c} \beta K_v \end{aligned}$$

So, static velocity error constant increased

7.5 Lag-Lead compensation

lead network;
  speeds up the response
  increase the stability

lag network;
  improves steady state

lead-lag network; lead network + lag network

Ex. 7-4

$$G(s) = \frac{4}{s(s+0.5)}$$

$$G_c(s) = K_c \frac{(s+\frac{1}{T_1})(s+\frac{1}{T_2})}{(s+\frac{\beta}{T_1})(s+\frac{1}{\beta T_2})}, \quad \beta > 1$$

desired location of dominants poles; $s = -2.50 \pm j4.33$
=>

$$G_c(s) = (10)(\frac{s+2.38}{s+8.34})(\frac{s+0.1}{s+0.0285})$$

$$G_c(s)G(s) = \frac{40(s+2.38)(s+0.1)}{(s+8.34)(s+0.0285)\ s\ (s+0.5)}$$

Closed loop poles; dominants: $s = -2.4539 \pm j\,4.3099$
others: s = -0.1003, s = -3.8604

zero s = -2.4 ; cause larger overshoot