Ch. 4 Mathematical Modeling of Fluid Systems and Thermal Systems

4-4 Hydraulic systems

operating pressure; 1~35 MPa, 70Mpa
combination of electrical and hydraulic systems

* Hydraulic Servo System

Fig.4.17 Hydraulic servo motor

Linearization of a hydraulic servo system

$$A_1 = A_3 = k(\frac{x_0}{2} + x)$$

$$A_2 = A_4 = k(\frac{x_0}{2} - x)$$

k : constant, γ : specific weight

$$q_1 = c_1 A_1 \sqrt{\frac{2g}{\gamma}(P_s - P_1)} = C_1 \sqrt{P_s - P_1}\,(\frac{x_0}{2} + x)$$

$$q_2 = c_2 A_2 \sqrt{\frac{2g}{\gamma}(P_s - P_2)} = C_2 \sqrt{P_s - P_2}\,(\frac{x_0}{2} - x)$$

$$q_3 = c_1 A_3 \sqrt{\frac{2g}{\gamma}(P_2 - P_o)} = C_1 \sqrt{P_2 - P_o}\,(\frac{x_0}{2} + x) = C_1 \sqrt{P_2}\,(\frac{x_0}{2} + x)$$

$$q_4 = c_2 A_4 \sqrt{\frac{2g}{\gamma}(P_1 - P_o)} = C_2 \sqrt{P_1 - P_o}\,(\frac{x_0}{2} - x) = C_2 \sqrt{P_1}\,(\frac{x_0}{2} - x)$$

$$q = q_1 - q_4 = C_1 \sqrt{P_s - P_1}\,(\frac{x_0}{2} + x) - C_2 \sqrt{P_1}\,(\frac{x_0}{2} - x)$$

$$q = q_3 - q_2 = C_1 \sqrt{P_2}\,(\frac{x_0}{2} + x) - C_2 \sqrt{P_s - P_2}\,(\frac{x_0}{2} - x)$$

incompressible fluid, symmetrical

$$q_1 = q_3\,,\ \ q_2 = q_4$$

$$P_s = P_1 + P_2\,,\ \ \triangle P \overset{\triangle}{=} P_1 - P_2$$

$$P_1 = \frac{P_s + \triangle P}{2}\,,\ \ P_2 = \frac{P_s - \triangle P}{2}$$

$$q = q_1 - q_4 = C_1 \sqrt{\frac{P_s - \triangle P}{2}} \left(\frac{x_0}{2} + x\right) - C_2 \sqrt{\frac{P_s + \triangle P}{2}} \left(\frac{x_0}{2} - x\right) \quad => f(x, \triangle P)$$

$$q - \bar{q} = a(x - \bar{x}) + b(P - \triangle \bar{P})$$

$$\bar{q} = f(\bar{x}, \triangle \bar{P})$$

valve coefficients;

$$a = \frac{\partial f}{\partial x}\Big|_{\bar{x}, \triangle \bar{p}}$$

$$= C_1 \sqrt{\frac{P_s - \triangle \bar{P}}{2}} + C_2 \sqrt{\frac{P_s + \triangle \bar{P}}{2}}$$

$$b = \frac{\partial f}{\partial \triangle p}\Big|_{\bar{x}, \triangle \bar{p}}$$

$$= -\left[\frac{C_1}{2\sqrt{2}\sqrt{P_s - \triangle \bar{p}}}\left(\frac{x_o}{2} + \bar{x}\right) + \frac{C_2}{2\sqrt{2}\sqrt{P_s + \triangle \bar{p}}}\left(\frac{x_o}{2} - \bar{x}\right)\right] < 0$$

Normal operating point;

$\bar{x} = 0$, $\triangle \bar{p} = 0$, $\bar{q} = 0$

Linearized mathematical model

$$q = K_1 x - K_2 \triangle P \qquad (4.27)$$

$$= (C_1 + C_2)\sqrt{\frac{P_s}{2}}\, x - (C_1 + C_2)\frac{x_0}{4\sqrt{2}\sqrt{P_s}} \triangle P$$

$$q = A_p \, \rho \dot{y}$$

$$A_p \triangle p = m\ddot{y} + b\dot{y}$$

4-5 Thermal systems

Fig.4.26

(1) temp of inflowing liquid: constant

hi ; small change in heat input rate

heat balance;

$$Cd\theta = (h_i - h_o)dt$$

$$C\frac{d\theta}{dt} = h_i - h_o$$

$$R = \frac{\theta}{h_o} = \frac{1}{Gc}$$

$$RC\frac{d\theta}{dt} + \theta = Rh_i \quad => \quad \frac{\Theta(s)}{H_i(s)} = \frac{R}{RCs+1}$$

(2) temp of inflowing liquid is changed : $\theta(i)$

heat input rate H, liquid flow rate G (kg/s); constant

heat output flow rate change ; ho

heat balance;

$$Cd\theta = (Gc\theta(i) - h_o)dt$$

$$RC\frac{d\theta}{dt} + \theta = \theta(i) \quad => \quad \frac{\Theta(s)}{\Theta_i(s)} = \frac{1}{RCs+1}$$

(3) temp of inflowing liquid, heat input rate; change

liquid flow rate ; constant

$$RC\frac{d\theta}{dt} + \theta = \theta(i) + Rh_i$$